

INCORPORATION OF THE SIX DIMENSIONAL
LINEARIZED EQUATIONS OF MOTION WITH A VARIABLE
STABILITY FLIGHT SIMULATOR

Merlin Richard Huckemeyer

United States Naval Postgraduate School



THESIS

INCORPORATION OF THE SIX DIMENSIONAL,
LINEARIZED EQUATIONS OF MOTION WITH A VARIABLE
STABILITY FLIGHT SIMULATOR

by

Merlin Richard Huckemeyer

Thesis Advisor:

D. M. Layton

September 1971

T142509

Approved for public release; distribution unlimited.

Incorporation of the Six Dimensional,
Linearized Equations of Motion with a Variable
Stability Flight Simulator

by

Merlin Richard Huckemeyer
Captain, United States Marine Corps
B.S.E.E., Ohio University, 1965

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1971

ABSTRACT

In this report the equations of motion for the six degrees of freedom of an aircraft were solved by an analog computer circuit. These solutions were incorporated with the instrumentation of a fixed base flight simulator to provide a variable stability aircraft demonstration device.

TABLE OF CONTENTS

I.	INTRODUCTION -----	5
II.	THE COMPUTER-SIMULATOR ARRANGEMENT -----	7
III.	AIRCRAFT AND AERODYNAMIC EQUATIONS OF MOTION ----	11
IV.	ANALOG SOLUTIONS -----	16
V.	FLIGHT SIMULATOR MODIFICATIONS -----	26
VI.	CONCLUSIONS -----	33
	LIST OF REFERENCES -----	35
	INITIAL DISTRIBUTION LIST -----	36
	FORM DD 1473 -----	37

ACKNOWLEDGEMENT

The author wishes to thank and express his appreciation for the assistance offered by: his wife, Rosalie, for her typing and understanding, Mr. Walt Landaker of the Electrical Engineering Department for his technical assistance, and Associate Professor D. M. Layton for his guidance as thesis advisor.

I. INTRODUCTION

This report is the fourth in a series of projects to convert a "Jet Propelled Aircraft Instrument Flying Trainer, Type C-11" manufactured by Link Aviation, Inc., Binghamton, New York, to a variable stability fixed base aircraft simulator. The objective being the provision of a dynamic real world demonstration device for use by the Aeronautics Department at the Naval Postgraduate School.

The first report [Ref. 1] considered the feasibility of converting this C-11 for use as a variable stability simulator. It was concluded that the computer system available could be made compatible with the simulator if some extensive modifications were made to the Link.

The second report [Ref. 2] concerned the physical alterations to the trainer and the replacement circuits necessary for simulator--computer compatibility. The overall dimensions of the C-11 were reduced and the A-C computer components were removed and replaced by D-C servo systems.

The third report [Ref. 3] integrated the longitudinal equations of motion, as solved on the analog computer, with the simulator controls and instrument displays. The conclusions of this third report stated that the quality of the flight characteristics of the trainer were not satisfactory if the simulator was to be useful as a demonstration device.

The goals of the present fourth project were twofold. The first objective was to improve the flying qualities of the simulator itself. This entailed improving or modifying the instrumentation of the simulator so that it would faithfully represent the computed solutions. The second objective was to integrate the full six degrees of motion into the computer solution and hence provide a complete flight envelope.

II. THE COMPUTER-SIMULATOR ARRANGEMENT

The hybrid computer system available for use with the simulator consists of a ComCor CI-5000 Hybrid Analog Computer, an SDS-9300 Digital Computer, two Adage AGT-10 graphics display units and various input output devices, (Figure 1). The computer systems are operated and maintained by the Electrical Engineering Department and the cockpit simulator by the Aeronautics Department at the Naval Postgraduate School.

The SDS-9300 (Scientific Data Systems Corp, El Segundo, California) has a 32k 24-bit core memory, two million 6-bit character drum, two magnetic tape units, line printer output, and multiple input peripherals. Its application to the real time solutions required for Link-computer interaction is enhanced by three features: a priority interrupt structure and external-word parallel input/output channel to the CPU, and a second port to memory which allows memory access at a 500 Khz word rate without degrading central processor operation.

The ComCor CI-5000 (ComCor Inc., Anaheim, California) as presently installed has 24 Summing Amplifiers, 28 Combination Amplifiers (Summer or Integrator), 10 Multipliers, 1 Resolver, 48 Servo Set Potentiometers, 32 Manual Set Potentiometers and other common analog components, as well as a limited parallel digital logic computer with "and" gates, flip flops, counters, and multiple clocks. The

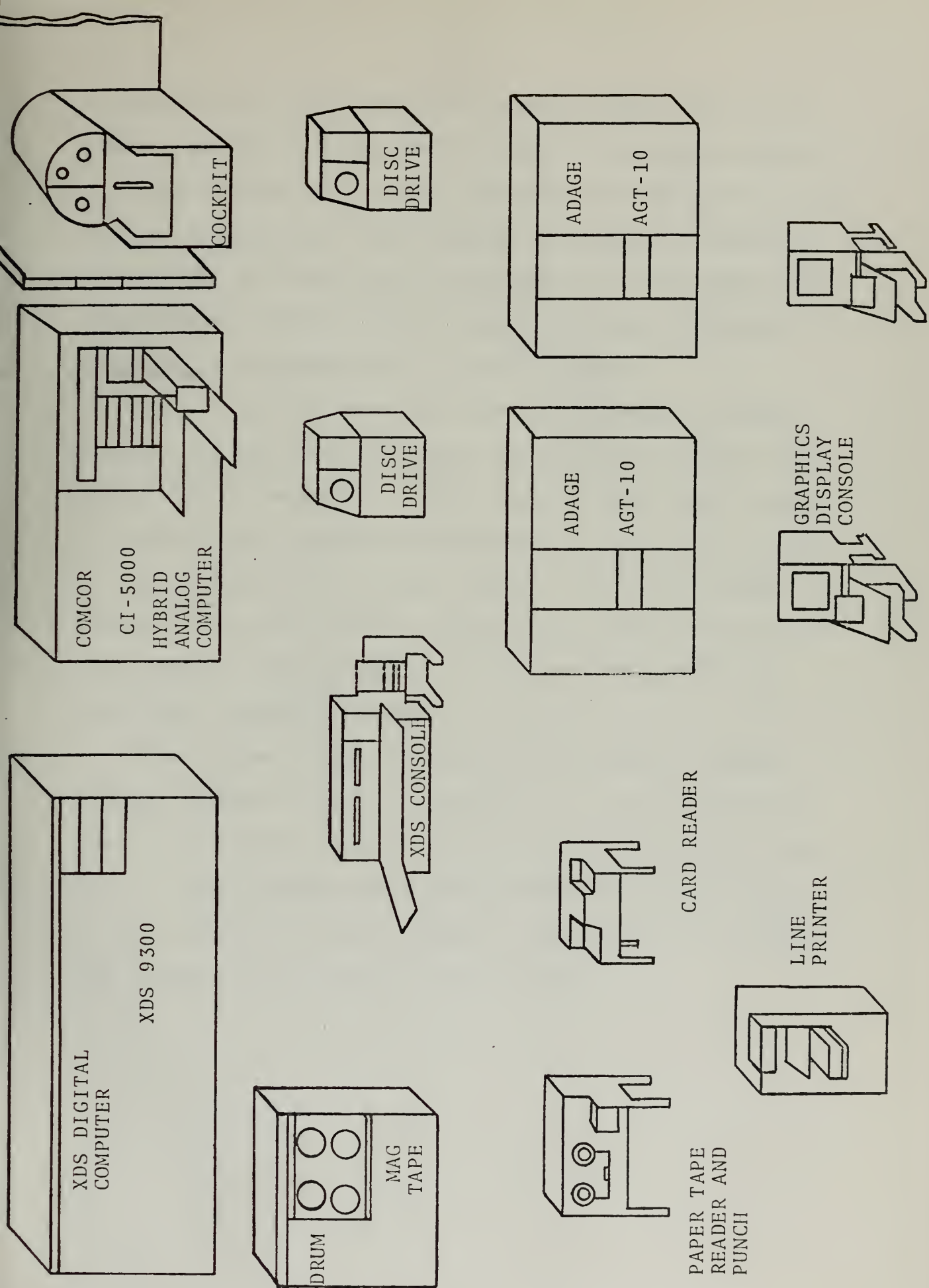


Figure 1. Computer System.

CI-5000 has a serial character-oriented adder and control system specifically designed for digital computer control.

The CI-5000 was the only computer used directly in the problem solution for this project, although the SDS-9300 was used to set the servo potentiometers. Meaningful incorporation of the graphics display unit into a hybrid solution is recommended as a future project.

Trunklines are hardwired from the simulator to the CI-5000 and may be patched into the problem solution from terminals on the analog board. Each of the control inputs is taken from a tapped potentiometer in the Link and sent to the computer as a voltage. And in turn the voltage outputs from the computer solutions are sent to the Link where they are transformed to visual readouts on the appropriate cockpit instruments.

The system originally required at least two people, in two different rooms, to operate it. One was needed to start and stop the computer and the other to fly the simulator. This inconvenience was overcome by using a logic circuit (Figure 2) that allowed the computer to be started and stopped from a switch in the cockpit.

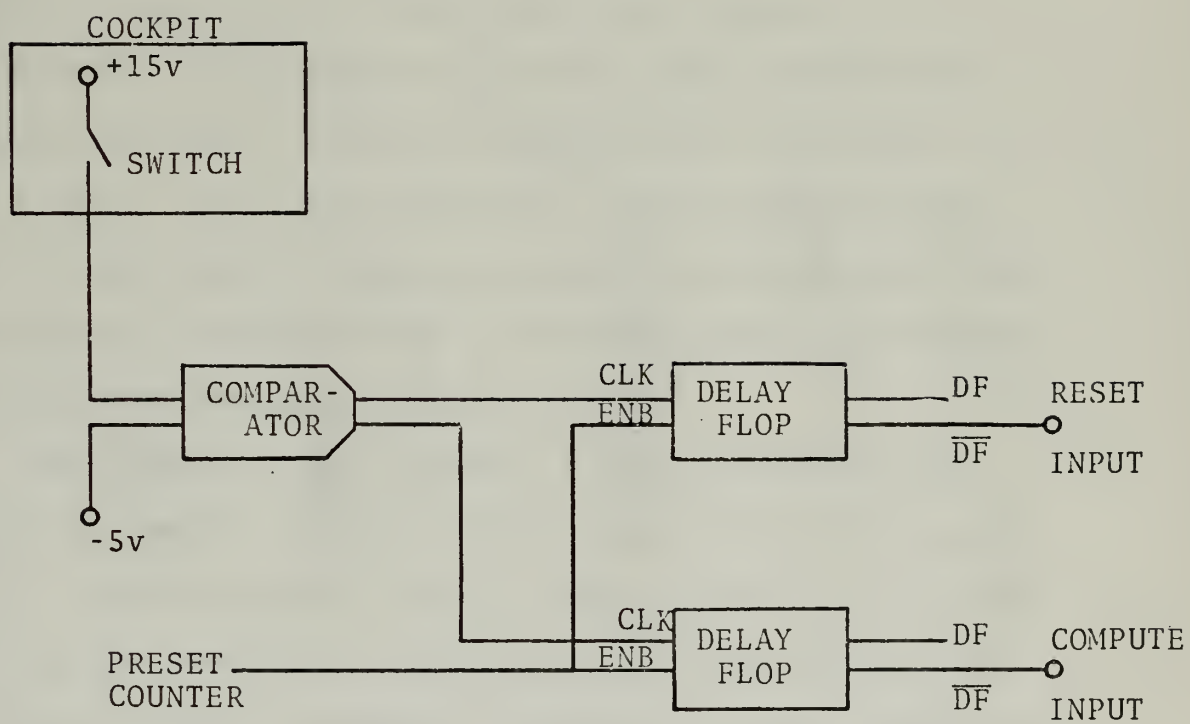


Figure 2. Fly-Reset Logic Circuit.

III. AIRCRAFT AND AERODYNAMIC EQUATIONS OF MOTION

The stability and body axes systems used to describe the force and moment equations were determined from the data and information available prior to this project. However, a brief description of these systems will be given for easy reference.

The body axes system is located at the center of gravity of the aircraft and fixed with respect to it. The mutually perpendicular axes have the conventional orientation with x being the longitudinal direction, y the lateral, and z the vertical. The determination of whether the longitudinal axis parallels the fuselage axis, wing chord, or thrust line is arbitrary and was chosen to parallel the fuselage reference line.

The stability axes system is also located at the center of gravity of the aircraft and fixed with respect to it. However, the x axis is chosen so that it is positive in the direction of motion of the aircraft with respect to a steady, symmetric, reference flight condition.

A graphical illustration of the axis system is shown in Figure 3. For this report, the x_b and x_s axes will be assumed parallel for the chosen reference flight condition.

The reference angles are the classical Euler Angles and are shown in Figure 3. In terms of the body axes the angular rates are given in Figure 4.

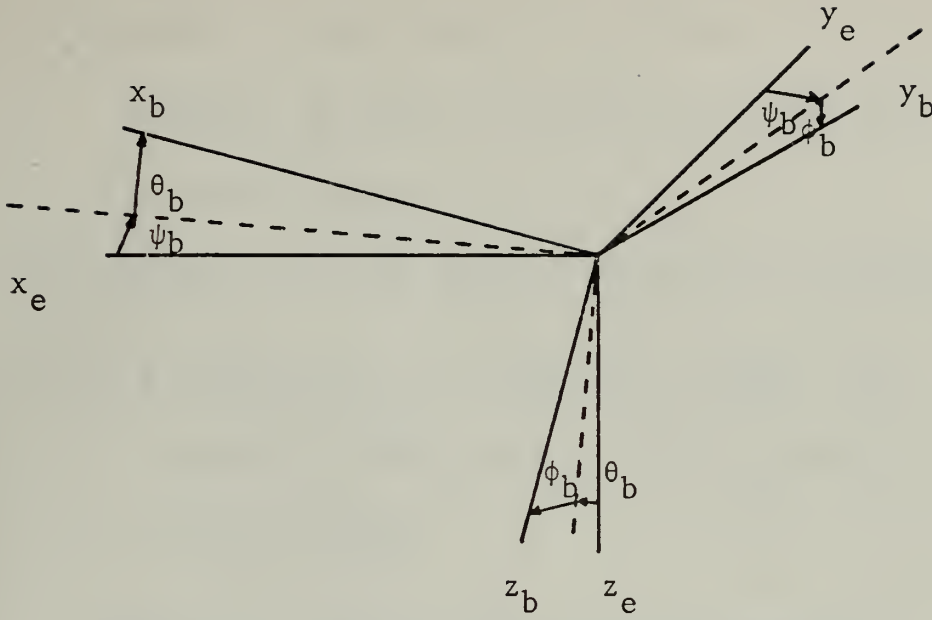


Figure 3. Euler Angles.

$$\dot{\phi}_b = p_b + \tan \theta_b (q_b \sin \phi_b + r_b \cos \phi_b)$$

$$\dot{\theta}_b = q_b \cos \phi_b - r_b \sin \phi_b$$

$$\dot{\psi}_b = \frac{q_b \sin \phi_b + r_b \cos \phi_b}{\cos \theta_b}$$

Figure 4. Angular Rates.

The equations of motion, as given in Reference 4, are listed in Figures 5 and 6. All moment equations are written with respect to the body axes and all force equations with respect to the stability axes.

$$L_b = I_x \dot{p}_b + (I_z - I_y) q_b r_b - I_{xz} (p_b q_b + \dot{r}_b)$$

$$M_b = I_y \dot{q}_b + (I_x - I_z) p_b r_b - I_{xz} (r_b^2 - p_b^2)$$

$$N_b = I_z \dot{r}_b + (I_y - I_x) p_b q_b - I_{xz} (\dot{p}_b - r_b q_b)$$

Figure 5. Body Axes Moment Equations.

$$\begin{aligned}
X_S &= m [\dot{u}_S - v_S (r_b \cos \alpha - p_b \sin \alpha) \\
&\quad + g(\sin \theta_b \cos \alpha - \cos \theta_b \cos \phi_b \sin \alpha)] \\
&\quad - T \cos (\alpha + \alpha_T) \\
Y_S &= m [\dot{v}_S + u_S (r_b \cos \alpha - p_b \sin \alpha) - g \cos \theta_b \sin \phi_b] \\
Z_S &= m [v_S (p_b \cos \alpha + r_b \sin \alpha) - u_S (q_b - \dot{\alpha}) \\
&\quad - g(\cos \theta_b \cos \phi_b \cos \alpha + \sin \theta_b \sin \alpha)] \\
&\quad - T \sin (\alpha + \alpha_T).
\end{aligned}$$

Figure 6. Stability Axes Force Equations.

As noted under the computer description, only one resolver is currently available on the CI-5000; therefore, small angle approximations are essentially required to handle the axis rotation problem. Further simplifications based on the small angle limitations are listed in Figure 7. (Reference 4)

- 1) α, β, θ are small [$\cos \alpha \approx 1, \sin \alpha \approx \alpha$]
- 2) Products and squares among α, β, p, q, r are negligible
- 3) $u_S \approx V \quad \tan \beta \approx \beta \quad \frac{v_S}{u_S} \approx \beta$
- 4) Thrust Component $T \sin (\alpha + \alpha_T)$ negligible.

Figure 7. Simplifications.

Substitution of these approximations into the force and moment equations, and considering the Euler angle rates, leads to the simplified equations listed in Figure 8.

MOMENTS:

$$\frac{L_b}{I_x} = \dot{p}_b - \frac{I_{xz}}{I_x} \dot{r}_b$$

$$\frac{M_b}{I_y} = \dot{q}_b$$

$$\frac{N_b}{I_z} = \dot{r}_b - \frac{I_{xz}}{I_z} \dot{p}_b$$

FORCES:

$$\dot{v} = \frac{X_s}{m} - g(\theta - \alpha \cos \phi_b) + \frac{T}{m}$$

$$v(\dot{\beta} + \dot{r}_b - p_b \alpha) = \frac{Y_s}{m} - g \sin \phi_b$$

$$v(\dot{\alpha} - \dot{q}_b) = \frac{Z_s}{m} + g \cos \phi_b$$

EULER ANGLE RATES:

$$\dot{\phi}_b = p_b + \theta_b \dot{\psi}_b$$

$$\dot{\theta}_b = q_b \cos \phi_b - r_b \sin \phi_b$$

$$\dot{\psi}_b = q_b \sin \phi_b + r_b \cos \phi_b$$

HEIGHT:

$$\dot{h} = v(\theta_b - \alpha \cos \phi_b - \beta \sin \phi_b)$$

$$\Delta v = \int \dot{v} dt$$

Figure 8. Simplified Equations.

The aerodynamic equations of motion for the T-33 aircraft are given in Reference 4 and listed in Figure 9.

AERODYNAMIC MOMENT EQUATIONS - BODY AXES

$$L_b/I_x = L_\beta \beta + \frac{\partial L_\beta}{\partial \alpha} \alpha \beta + L_p p + L_r r + \frac{\partial L_r}{\partial \alpha} \alpha r$$

$$+ L_{\delta_A} \delta_A + L_{\delta_R} \delta_R$$

$$N_b/I_z = N_\beta \beta + N_p p + \frac{\partial N_p}{\partial \alpha} \alpha p + N_r r + N_{\delta_A} \delta_A$$

$$+ N_{\delta_R} \delta_R + \frac{\partial N_{\delta_A}}{\partial \alpha} \alpha \delta_A$$

$$M_b/I_y = M_\alpha \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_q q_b + M_{\delta_E} \delta_E$$

AERODYNAMIC FORCE EQUATIONS - STABILITY AXES

$$X_s = - q_0 S [C_{D_0} + C'_{D_\alpha} \alpha + C'_{D_{\alpha^2}} \alpha^2]$$

$$- q_0 S \left(\frac{2}{v_0} \right) [C_{D_0} \Delta v + C'_{D_\alpha} \alpha \Delta v + C'_{D_{\alpha^2}} \alpha^2 \Delta v]$$

$$Y_s = m v [Y_\beta \beta + Y_{\delta_R} \delta_R]$$

$$Z_s = - q_0 S [C_{L_0} + C_{L_\alpha} \alpha + C_{L_{\delta_E}} \delta_E]$$

$$- q_0 S \left(\frac{2}{v_0} \right) [C_{L_0} \Delta v + C_{L_\alpha} \alpha \Delta v]$$

Figure 9. Aerodynamic Equations.

IV. ANALOG SOLUTIONS

The physical data for the constants and derivatives of the T-33 aircraft, as given in Reference 4, were inserted in the listed equations. The scaled analog computer circuit solutions were then determined as shown in Figures 10 through 13. The potentiometer values are collected and listed in Table I.

An interface network is shown in Figure 14 with the potentiometer values included in Table I. These added circuits were necessary to make the scaled computer variables compatible with the separately scaled simulator inputs and control outputs.

In the preceding report (Reference 3), the longitudinal equations, as developed in Reference 4, were solved on the digital and analog computers. Both solutions produced the unacceptable aircraft response shown in Figure 15. Initially, the same results were obtained for this study. To get the response shown in Figure 16, the coefficient of drag (C_{D_0}) in X_s was multiplied by ten to damp the long period oscillation, M_q was multiplied by one hundred to damp the short period oscillation, M_α was multiplied by ten to increase the frequency of the short period oscillation, and M_{δ_E} was multiplied by ten to maintain a response to a change in the elevator deflection (δ_E).

Although it appears that gross variations have been introduced in to certain stability parameters in order to

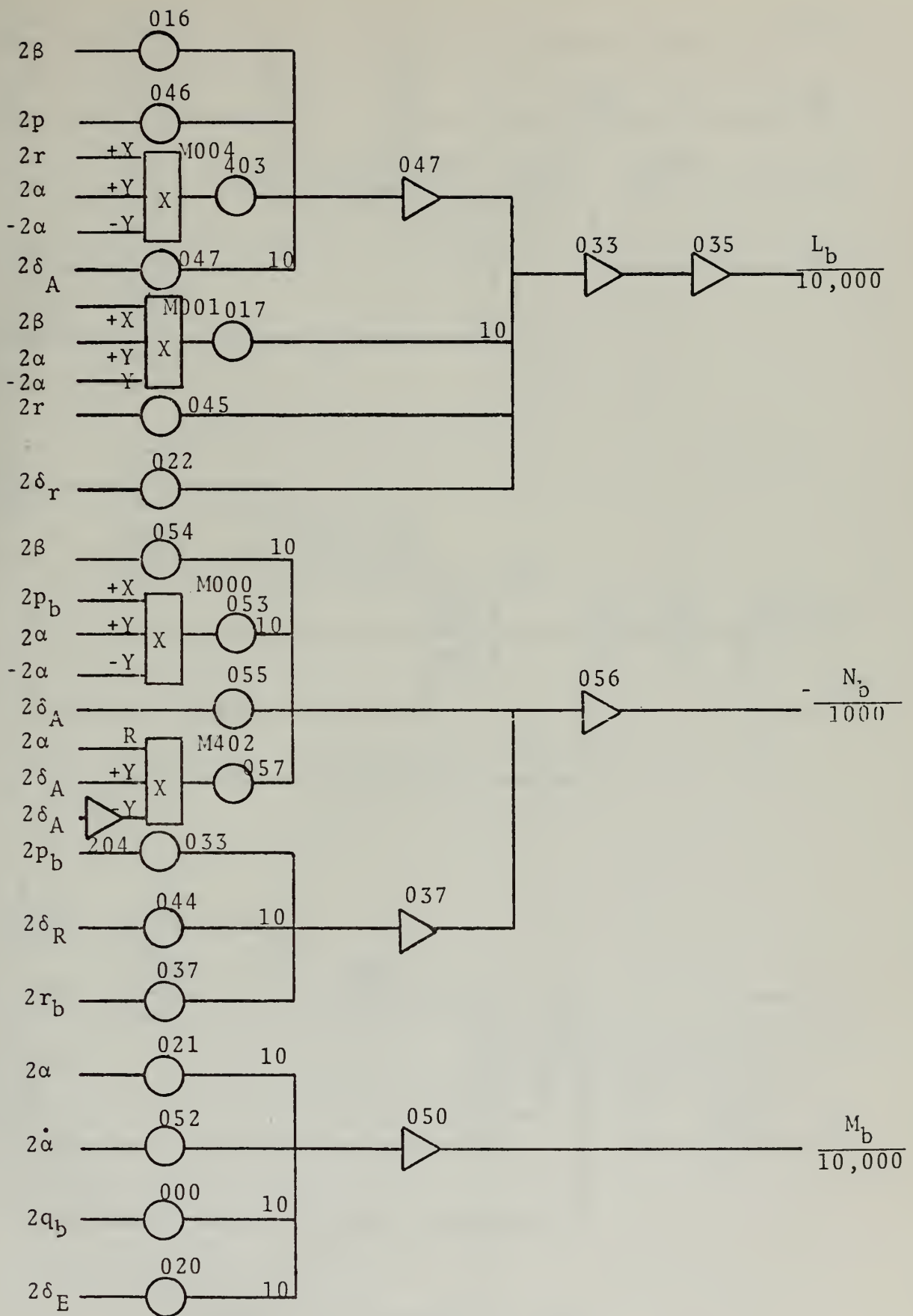


Figure 10. Analog Circuits.

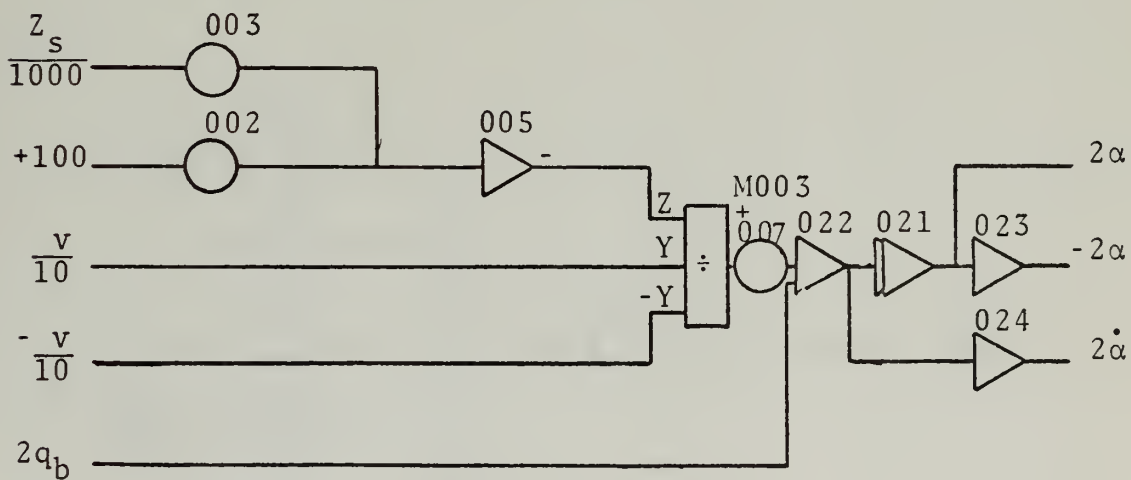
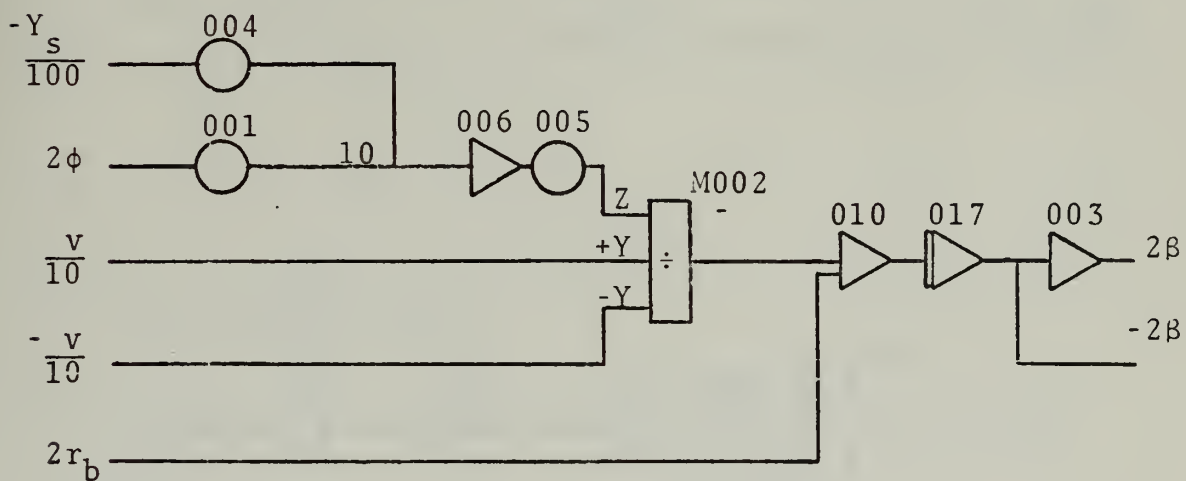
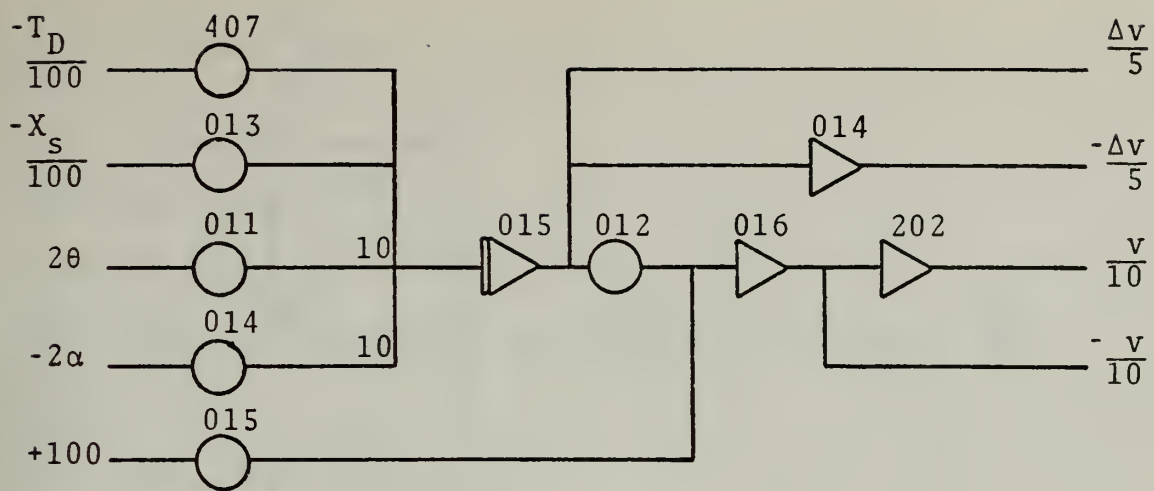


Figure 11. Analog Circuits.

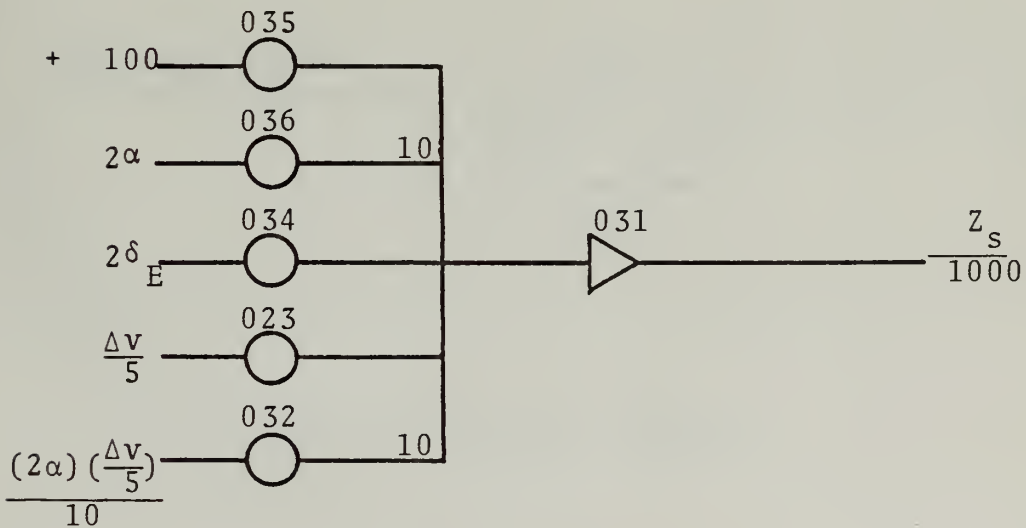
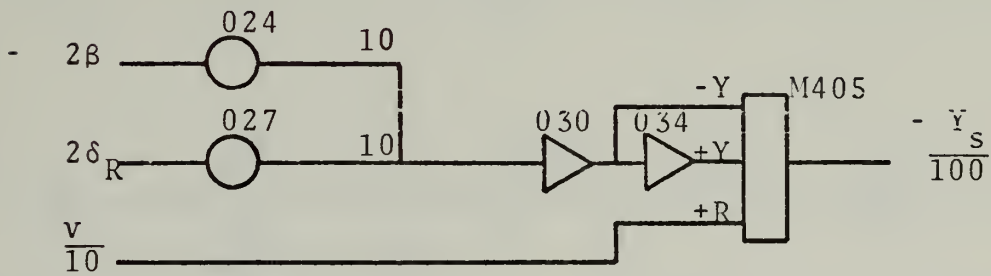
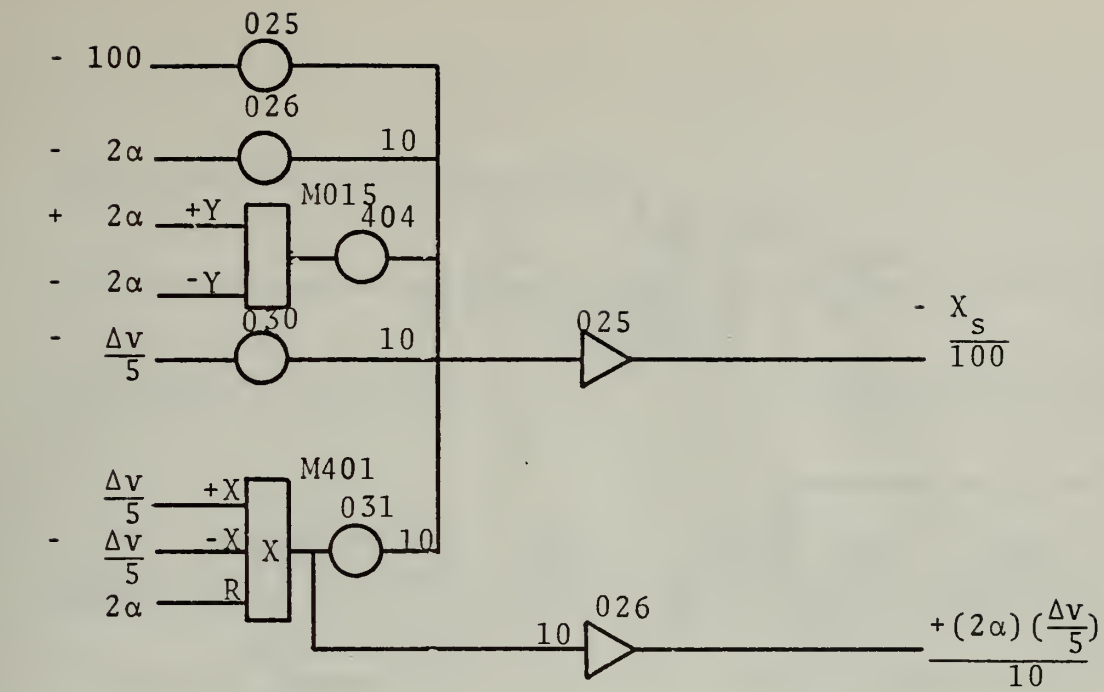


Figure 12. Analog Circuits.

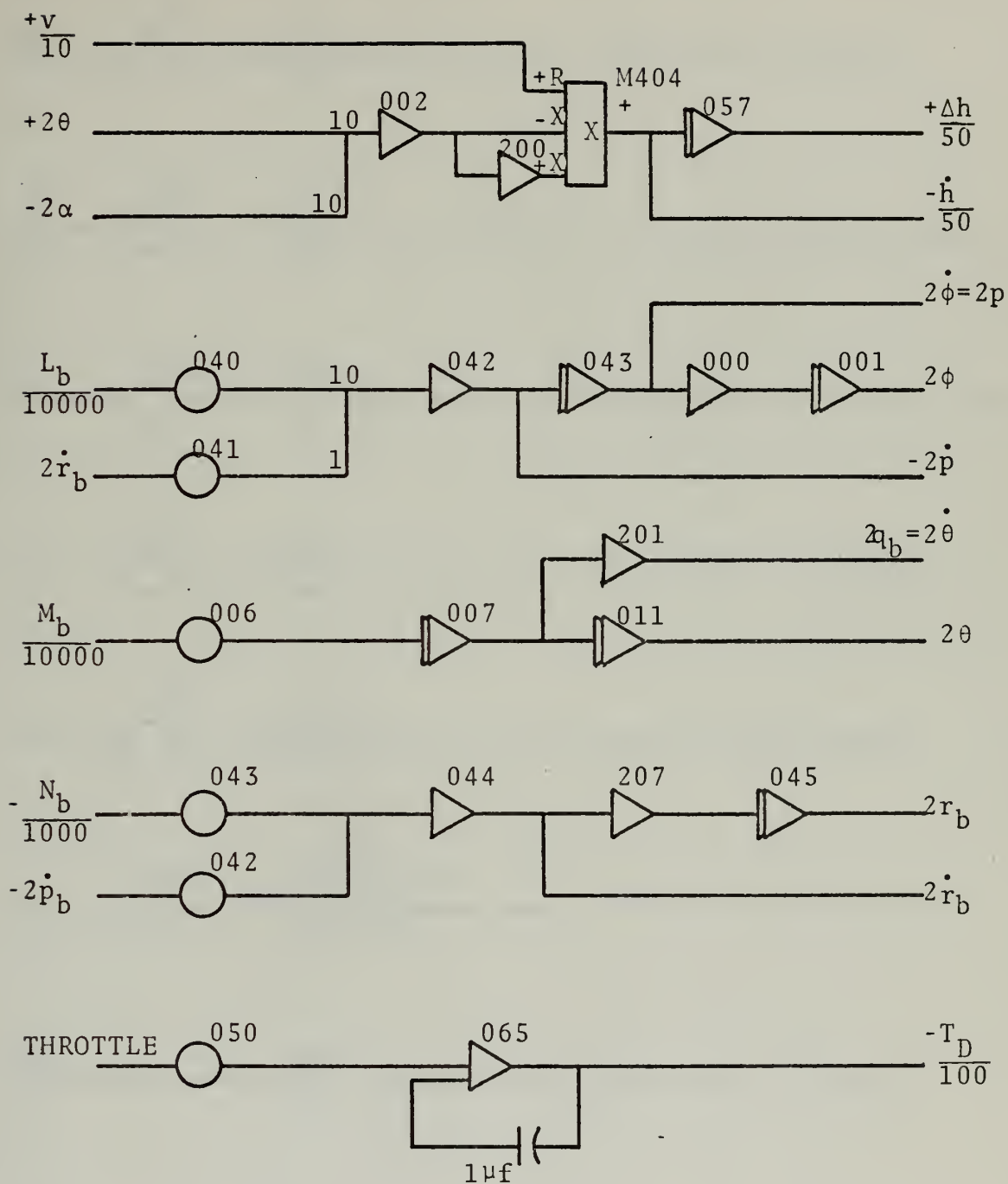


Figure 13. Analog Circuits.

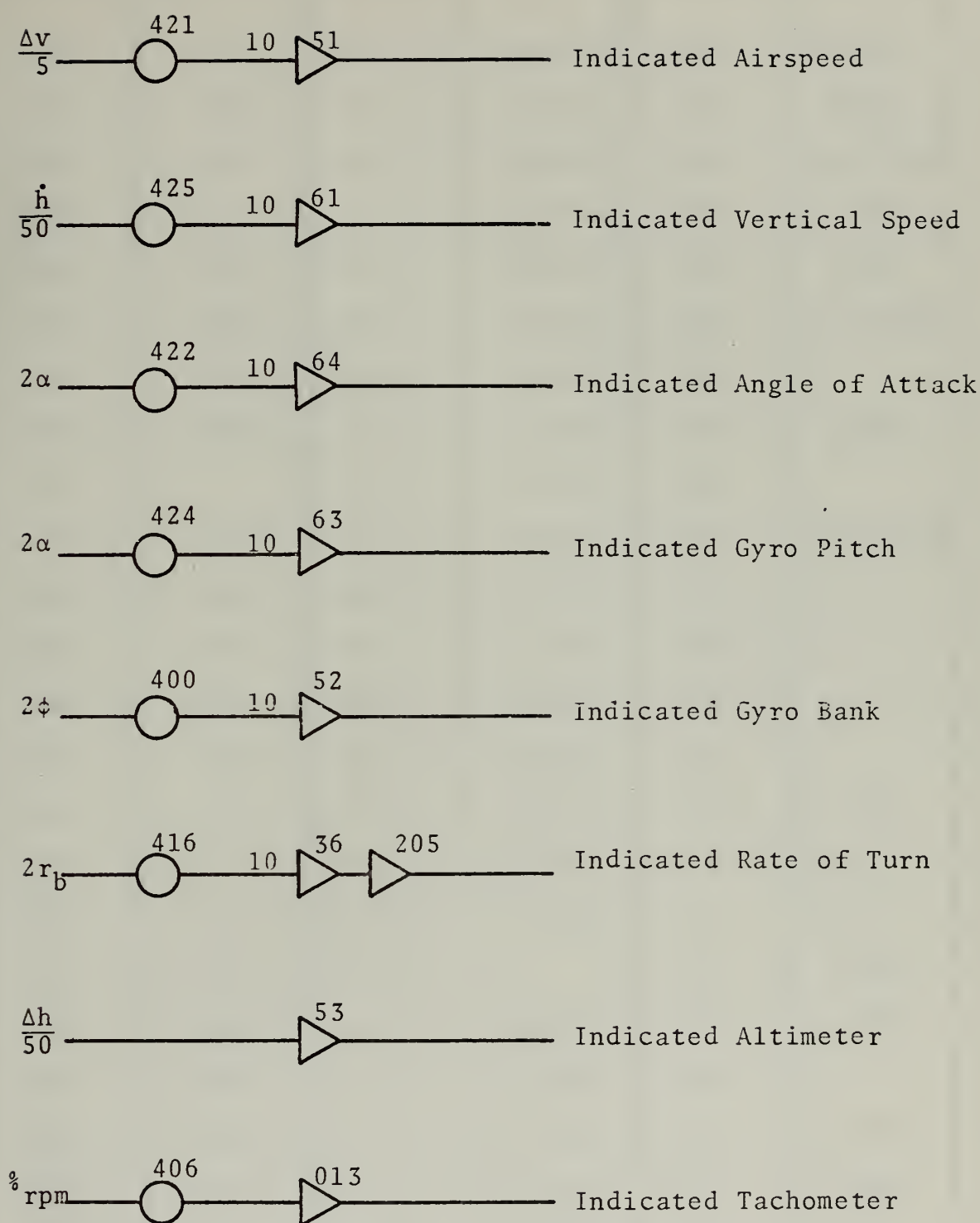


Figure 14. Interface Circuits.

POT NUMBER	SETTING	POT NUMBER	SETTING	POT NUMBER	SETTING
000	.1329	030	.1776	400	.1666
001	.3220	031	.1229	401	.9999
002	.0644	032	.0462	402	$\delta_A(.9000)$
003	.5192	033	.0102	403	.0466
004	.0519	034	.1604	404	.2168
005	.0100	035	.1319	405	$\delta_R(.0800)$
006	.9660	036	.2820	406	
007	.0100	037	.0858	407	.0361
010		040	.1170	410	
011	.3220	041	.0287	411	$\delta_E(.7200)$
012	.5000	042	.0135	412	
013	.0519	043	.0549	413	
014	.3220	044	.1207	414	
015	.6100	045	.0113	415	
016	.1433	046	.0313	416	.5500
017	.1137	047	.0172	417	ϕ_E
020	.2381	050	.2701	420	T_D
021	.1219	051		421	.5600
022	.0298	052	.0132	422	.3333
023	.2161	053	.0454	423	
024	.4704	054	.9720	424	.1111
025	.1084	055	.0397	425	.1650
026	.0750	056	.0500	427	ϕ_A
027	.1277	057	.6375	437	ϕ_R

TABLE I. Potentiometer Settings.

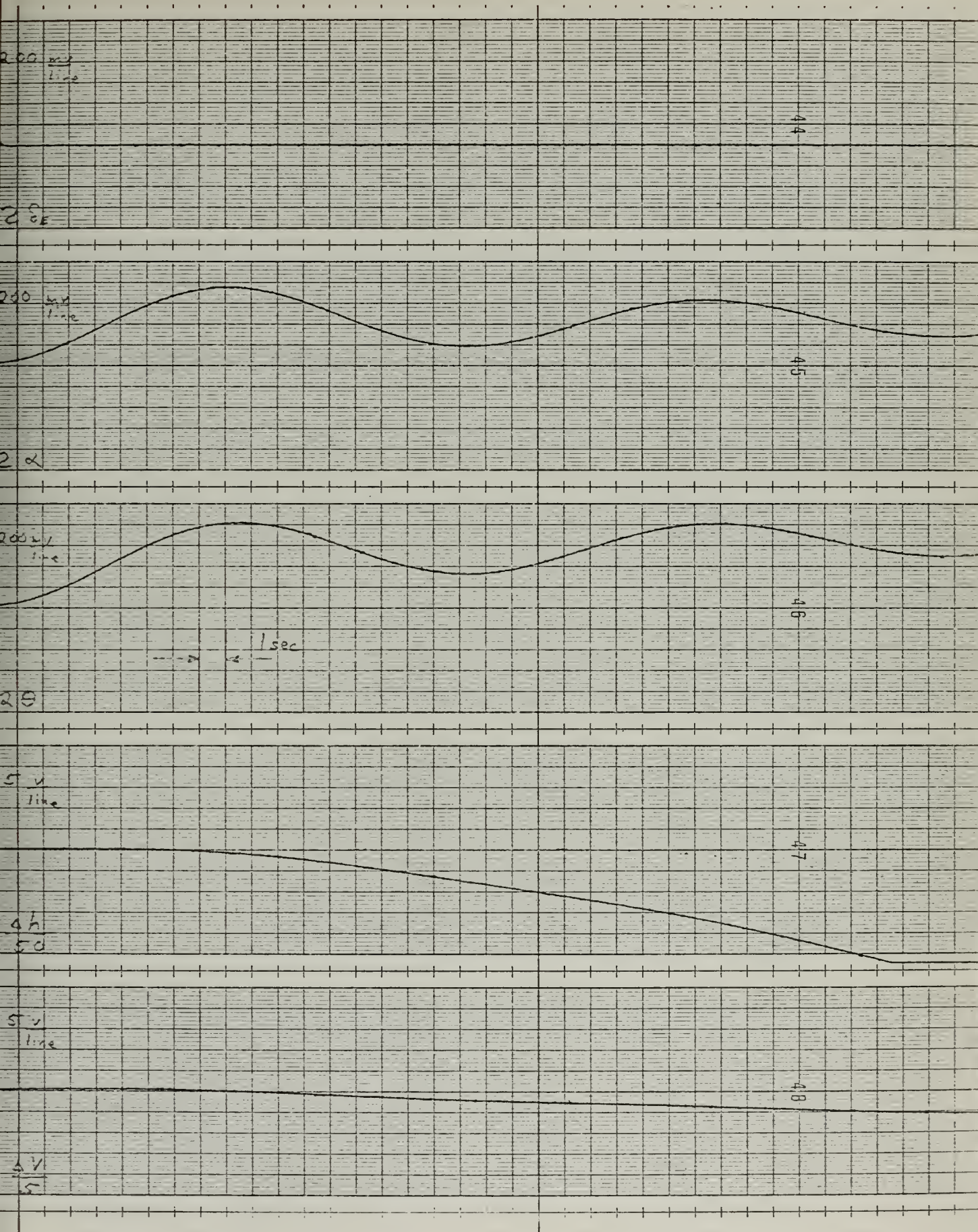


Figure 15. Initial Response.

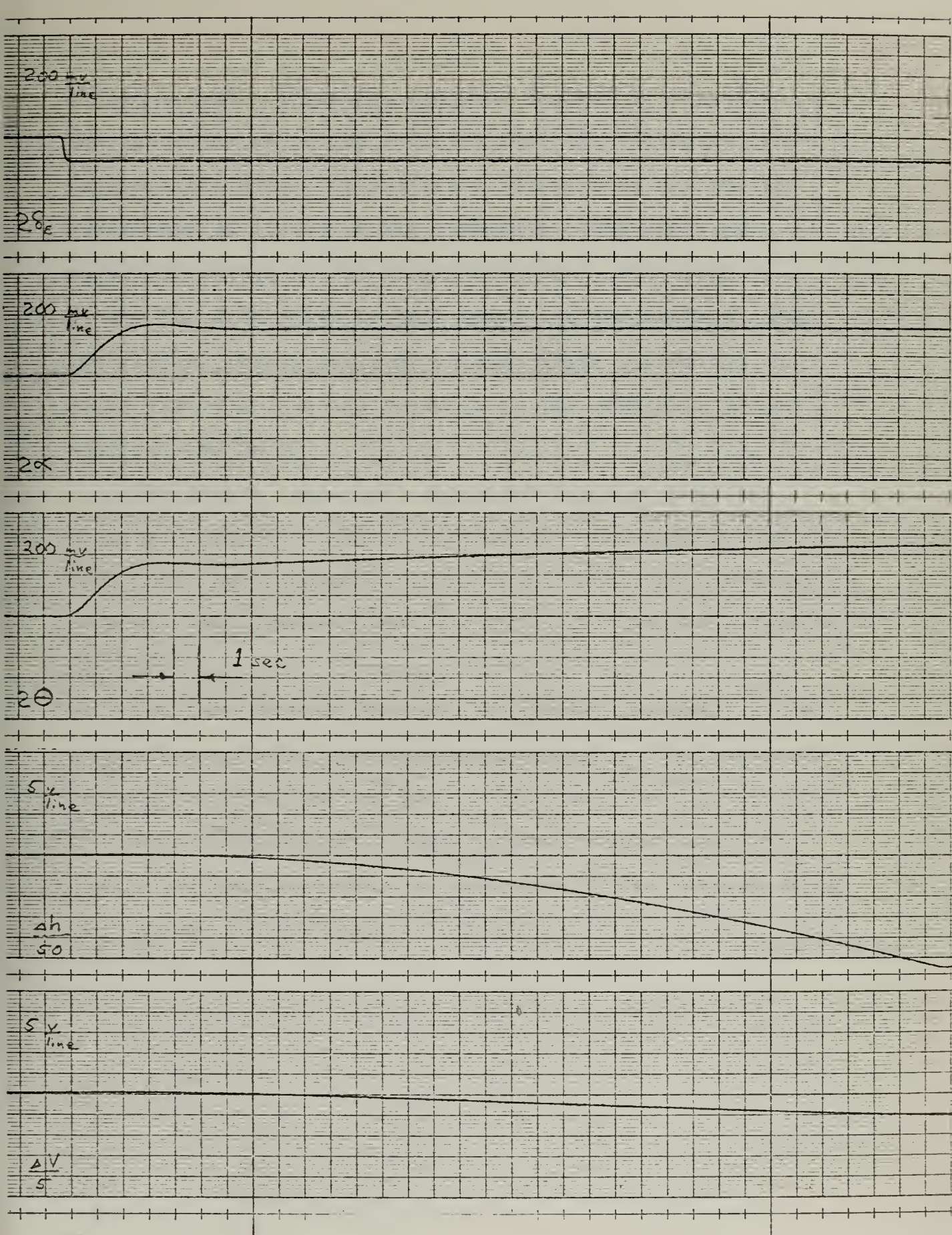


Figure 16. Modified Response.

obtain a stable solution, discrete digital solutions to the equations of motion indicate that the increased coefficients are nearer to predicted values than are the coefficients as listed in the reference. The order of magnitude change in the drag coefficient, for example, causes the drag value to fall nearly on the curve of drag coefficient versus angle of attack, while the values derived from the data and formulae of the reference do not approach the curve.

The lateral equations also proved to be unusable initially. Once an angle of bank (ϕ) was assumed, the yaw angle (β) would increase till computer overload. The solution was made stable by multiplying N_β , by ninety to get an acceptable frequency for the Dutch Roll mode.

The changes made were arbitrary, but necessary to get a useable solution from the data available. The circuits, as modified, were considered to represent the aircraft in its normal flight mode. Demonstrations that changed the stability characteristics of the aircraft used this mode as the reference condition.

V. FLIGHT SIMULATOR MODIFICATIONS

The C-11 Instrument flight simulator was not originally designed to represent any particular aircraft. The primary use was for instrument instruction, not as a variable stability demonstrator. Consequently, no provisions were made for selectively varying chosen performance parameters; hence, the previous projects proceeded to reconfigure the C-11 to convert it to a variable stability simulator.

With one exception, all the flight instruments had been converted to a dc-ac servo system. In these systems, a dc motor was driven by a differential amplifier that compared the input signal to a feedback signal from a follow up potentiometer. Motor movement was sensed through a gear train by an AC synchronous servo system that positioned the cockpit instrument. A block diagram representation is shown in Figure 17.

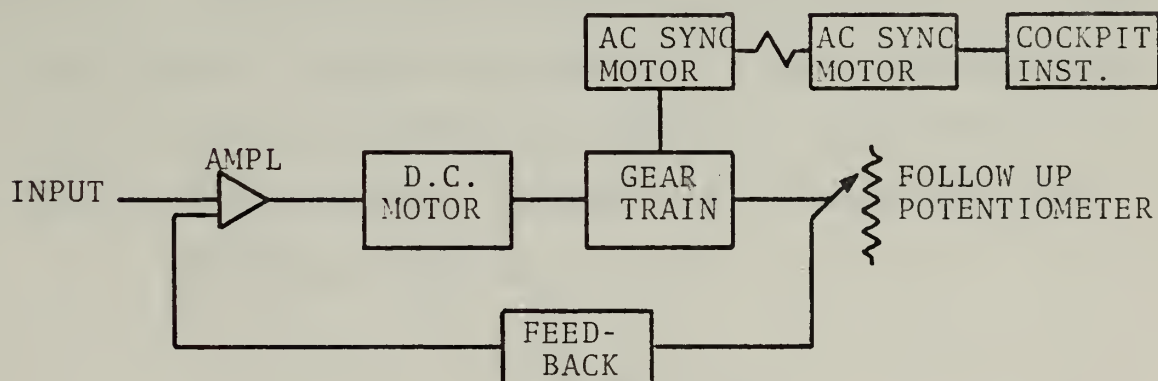


Figure 17. Block Diagram Instrument Drive System.

There were two basic problems with the system as designed and installed; it did not respond to small inputs and once it did respond, it would not re-center to the same position each time when the signal was removed. The two problems are directly related in that the dc drive motors need a finite voltage to respond to a signal, be it input or feedback. For the motors available, this voltage was $0.5\text{v} \rightarrow 4\text{v}$. With the cost of better dc motors prohibitive, it was decided to replace as many instruments as possible with direct current meter movements and use the best of the available motors for the remaining instruments.

Consequently the airspeed, angle of attack, vertical speed indicator, and accelerometer servo systems were replaced with dc meter movements. The indicator instruments were removed and the dials that the pilot sees were mounted on the new movements and replaced in the cockpit. Thus, the pilot still sees the conventional aircraft instrumentation. To provide for calibration, the output of a trunkline from the computer connected to a potentiometer which acts as a voltage divider and supplies an appropriate voltage to the indicator. (Figure 18.) A resistance was

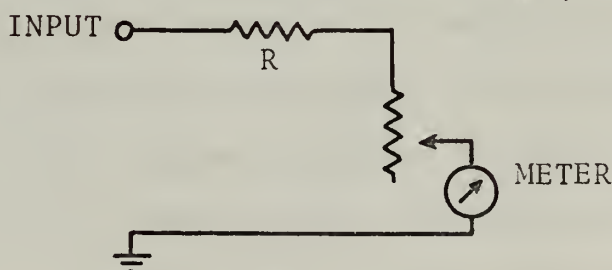


Figure 18. D.C. Meter Indicator System.

placed in series with the potentiometer to prevent inadvertently burning out the meter during the calibration process.

The altimeter circuit was modified from the original design, Figure 19, by replacing the single turn follow up potentiometer with a three turn potentiometer and then increasing the feedback gain to the amplifier. This arrangement (elimination of R_1 , R_2 , and C) provided a good centering characteristic; and also improved the response of the system as shown in the before and after response of the follow up potentiometer movement. (Figure 20.) The input resistor was also reduced to increase the allowable altitude variation from 6000 to 10,000 feet.

The gyro bank system was treated similarly to the altimeter. The response was improved by changing the motor and increasing the gain of the feedback from the follow up potentiometer. Although the response is not completely smooth, it is satisfactory for demonstration purposes.

The gyro pitch system was left as an AC servo system due to the size limitations on the motor (it mounts inside the gymbaled ball). The poor response of the original design was traced to the omission of a feedback path for the operational amplifier and improper phase relationships in the chopper-amplifier circuit (Figure 21). By inserting the feedback path and optimizing capacitor values, the problem was corrected and the circuit responds smoothly and accurately.

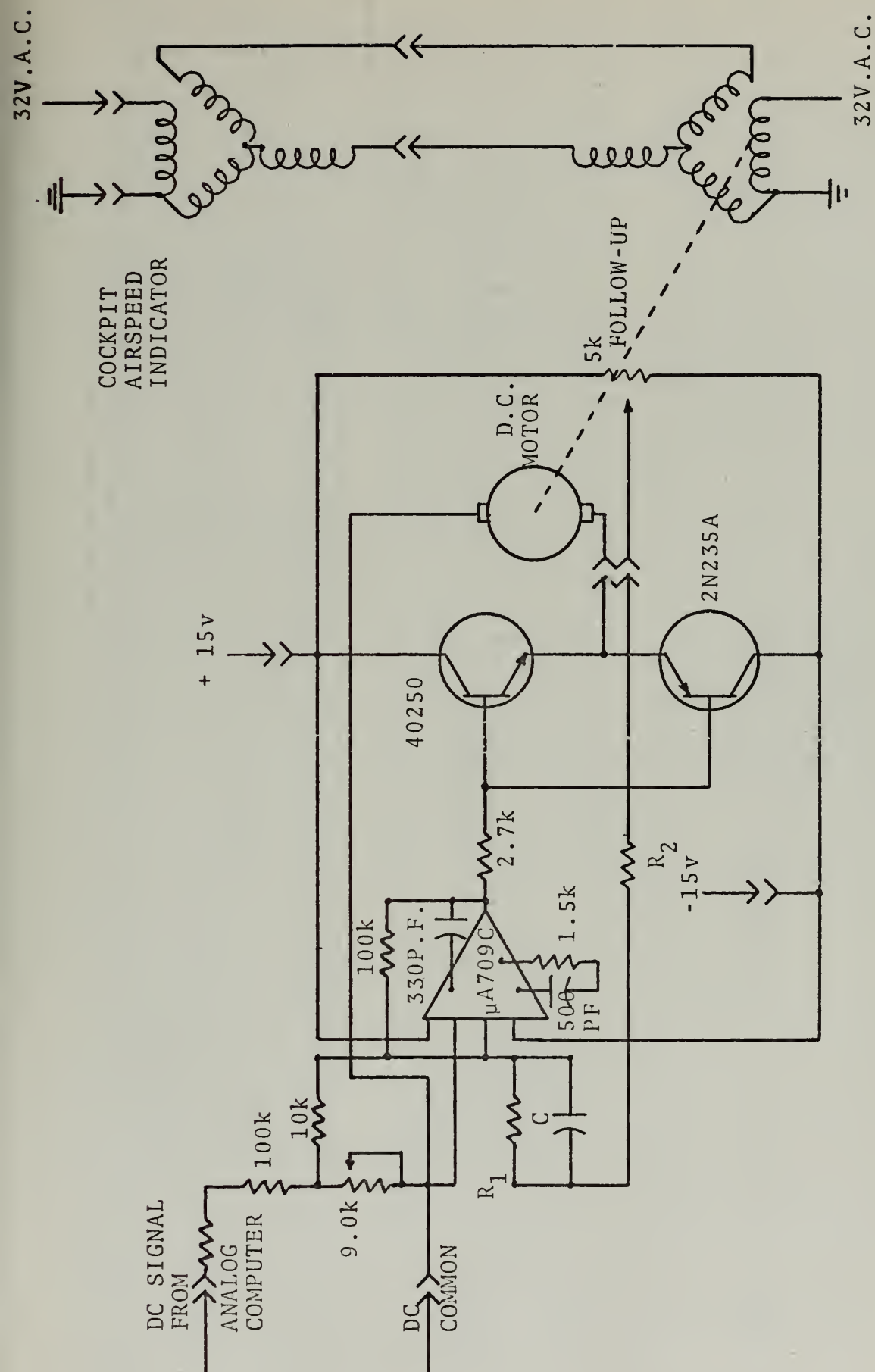


Figure 19. Modified Altimeter Indicator Drive Circuit.

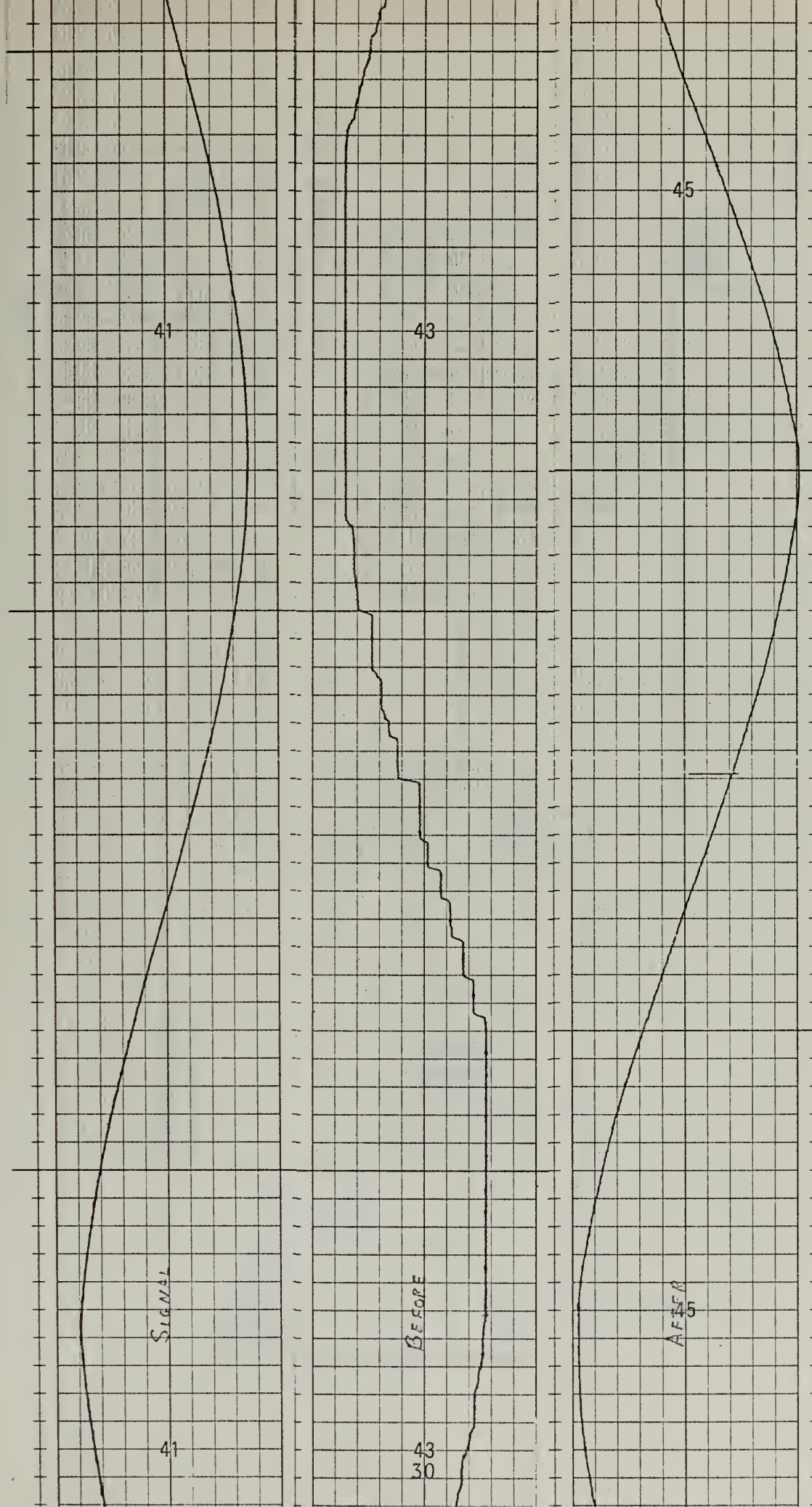


Figure 20. Servo Response Characteristics.

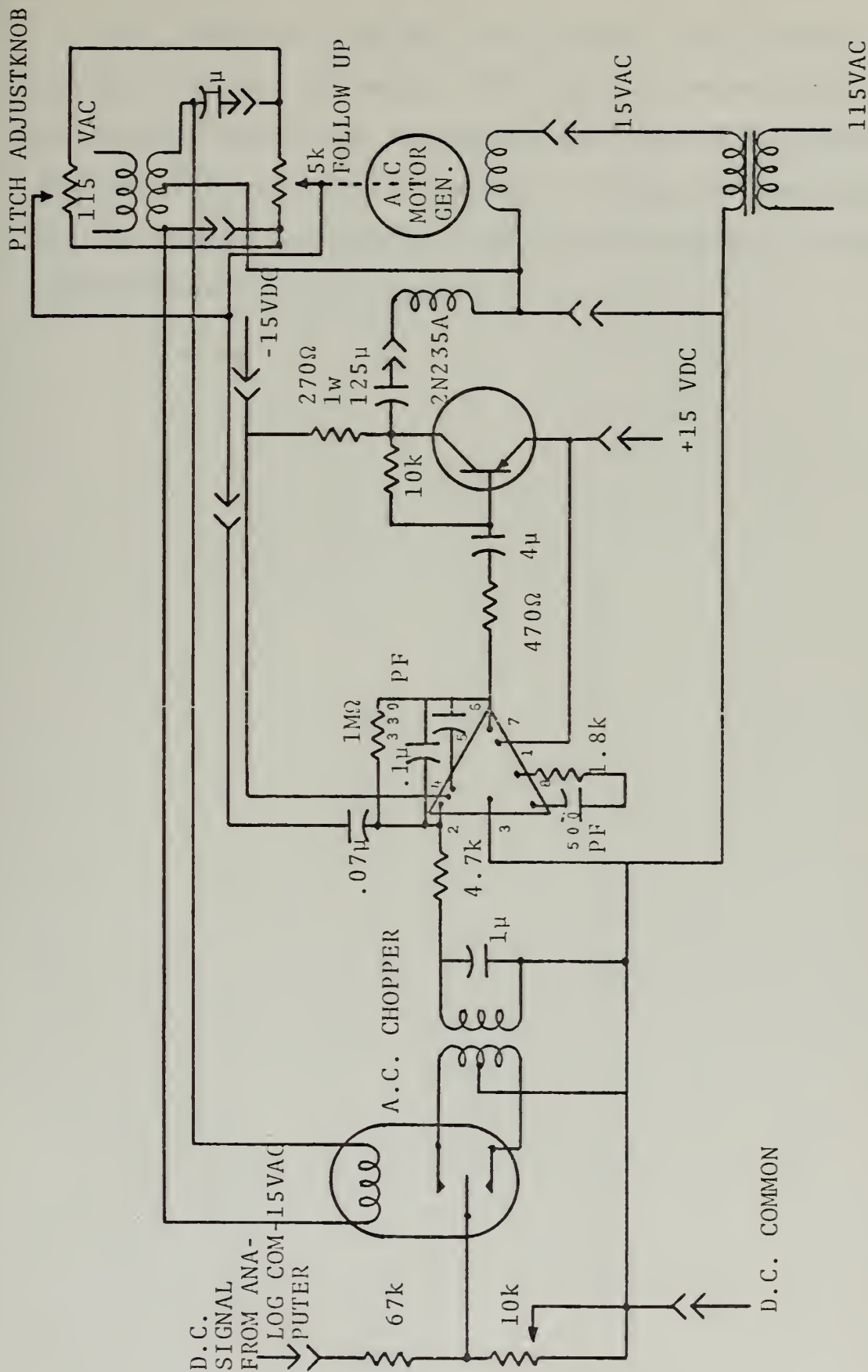


FIGURE 21. MODIFIED GYRO HORIZON (PITCH ONLY) DRIVE CIRCUIT.

The tachometer and gyro bank systems were treated similarly to the altimeter. The responses were improved by changing motors and increasing the gain of the feedback from the follow up potentiometers. Though the responses are not completely smooth, they are satisfactory for meaningful demonstration purposes.

VI. CONCLUSIONS

The stated goals of this project were to improve the simulator response and to incorporate the six degrees of freedom in the computer solution. Both objectives were accomplished but with varying degrees of success.

The instrumentation of the simulator now responds accurately and smoothly to the computer output. The control stick has a small dead space in the centered position, though this does not appear to significantly effect the pilot's response. The operation of the simulator is considered quite acceptable for variable stability demonstrations.

The solution of the equations of motion with the data given in Reference 4 are not considered acceptable. The reason for the arbitrary changes that were needed to utilize the solutions are not understood and make comparison between the simulator and the aircraft itself meaningless. However, changing stability parameters did change the responses of the simulator, though it did not appear that all of the reactions were in proportion to the changes made.

Consequently, it is recommended that for future work with the simulator, new aircraft data be obtained and that the aerodynamic equations be solved in accordance with that data. If the original solutions would correspond to the actual aircraft response, more confidence could be

placed in the varied responses due to changes in the aircraft stability parameters.

The ease with which aircraft parameters could be changed was demonstrated. This, combined with the ability to program a sequence of changes on the digital computer warrants further effort for this project.

LIST OF REFERENCES

1. Johnson, J. A., Study of a C-11 Instrument Flight Trainer as a Variable Stability Model and Simulator for Studies in Flight Dynamics, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1967.
2. Sweeney, C. J., Jr., Conversion of a C-11B Instrument Flight Trainer into a Variable Stability Flight Simulator, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1968.
3. Willetts, L. J., Jr., An Investigation of the Longitudinal Handling Qualities of a Variable Stability Flight Simulator, A.E. Thesis, Naval Postgraduate School, Monterey, California, 1969.
4. Cornell Aeronautical Laboratory Report TC-1921-F-2, A Functional Description and Working Data for the Variance-Stability System T-33 Airplane, by D. L. Key, p. 53-103, October 1965.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2. Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3. Chairman, Department of Aeronautics Naval Postgraduate School Monterey, California 93940	1
4. Professor Donald M. Layton Department of Aeronautics Naval Postgraduate School Monterey, California 93940	1
5. CAPT Merlin R. Huckemeyer, USMC 55 San Juan Road, Box 24 Salinas, California 93901	1

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

3. REPORT TITLE

INCORPORATION OF THE SIX DIMENSIONAL, LINEARIZED EQUATIONS
OF MOTION WITH A VARIABLE STABILITY FLIGHT SIMULATOR

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis; September 1971

5. AUTHOR(S) (First name, middle initial, last name)

Merlin Richard Huckemeyer

6. REPORT DATE

September 1971

7a. TOTAL NO. OF PAGES

38

7b. NO. OF REFS

4

8a. CONTRACT OR GRANT NO.

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned
this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

Naval Postgraduate School
Monterey, California 93940

13. ABSTRACT

In this report the equations of motion for the six degrees of freedom of an aircraft were solved by an analog computer circuit. These solutions were incorporated with the instrumentation of a fixed base flight simulator to provide a variable stability aircraft demonstration device.

DD FORM 1473 (BACK)
1 NOV 65
/N 0101-807-6821

Thesis

H842

c.1

Huckemeyer

130554

Incorporation of the
six dimensional, line-
arized equations of mo-
tion with a variable
stability flight simu-
lator.

Thesis

H842

c.1

Huckemeyer

130554

Incorporation of the
six dimensional, line-
arized equations of mo-
tion with a variable
stability flight simu-
lator.

thesH842

Incorporation of the six dimensional, li



3 2768 002 13197 1

DUDLEY KNOX LIBRARY